Implied Correlation Index: A new measure of diversification

Vasiliki D. Skintzi and Apostolos-Paul N. Refenes

Financial Engineering Research Center
Department of Management Science & Technology
Athens University of Economics and Business
Background

- Correlation estimates are required in most applications in finance (asset pricing models, capital allocation, risk management, option pricing and hedging).

- Several studies have documented the time-variability of correlation (Von Fustenberg and Jeon, 1989; Koch and Koch, 1991; Erb *et al*, 1994; Longin and Solnik, 1995).

- A wide variety of conditional correlation models exists.

- A main limitation of these approaches: they are based on a historical information set.
Motivation

- The market forecast of future correlation implied by option prices is an appealing measure and its properties have not been studied in the literature.

- Option prices have been widely used in order to obtain implied volatilities.

- Studies on the predictive ability of implied volatility found that the implied volatility provides incremental information on future volatility not included in historical forecasts.

- Only a limited number of studies has used option prices to derive implied correlation measures and only for currency options.
Our idea...

- We propose a general methodology for deriving implied correlation measures from option prices that can be applied to options on any portfolio of assets.

- By applying this methodology to index options, the derived implied correlation measure (called *Implied Correlation Index, CIX*) is a forecast of the future market portfolio diversification.
This Paper - Contributions

- A new methodology for constructing an implied correlation index from option prices is proposed.

- The proposed methodology is applied to the Dow Jones Average (DJIA) index option prices.

- The daily dynamics of the implied correlation index are examined.

- We explore whether several stylized facts about correlation are also present in the implied correlation index.

- Finally, the forecasting performance of implied correlations is assessed.
Outline of the talk

- CIX definition and interpretation
- Empirical Application: Data and Methodology
- Statistical Properties of CIX
- Stylized facts
- Forecasting performance
- Conclusions
Definition of the Implied Correlation Index

- The variance of a portfolio consisting of $N$ assets is the following:

$$\sigma_{P,t}^2 = \sum_{i=1}^{N} W_{i,t}^2 \sigma_{i,t}^2 + 2 \sum_{i=1}^{N-1} \sum_{j>i} W_{i,t} W_{j,t} \rho_{ij,t} \sigma_{i,t} \sigma_{j,t}$$

- The Implied Correlation Index (CIX) is defined as the correlation $\rho_t$ that if used instead of the $N(N-1)$ individual correlations will result to the same portfolio variance, i.e.

$$\sigma_{P,t}^2 = \sum_{i=1}^{N} W_{i,t}^2 \sigma_{i,t}^2 + 2 \sum_{i=1}^{N-1} \sum_{j>i} \rho_t W_{i,t} W_{j,t} \sigma_{i,t} \sigma_{j,t}$$

- CIX is a measure of the average level of correlation in the portfolio.

$$CIX_t = \sum_{i=1}^{N} \sum_{j>i} c_{ij,t} \rho_{ij,t} \quad where \quad c_{ij,t} = \frac{W_{i,t} W_{j,t} \sigma_{i,t} \sigma_{j,t}}{\sum_{i=1}^{N} \sum_{j>i} W_{i,t} W_{j,t} \sigma_{i,t} \sigma_{j,t}}$$
Construction of the Implied Correlation Index

\[ CIX_t = \frac{\sigma_{P,t}^2 - \sum_{i=1}^{N} W_{i,t} \sigma_{i,t}^2}{2 \sum_{i=1}^{N-1} \sum_{j>i} W_{i,t} W_{j,t} \sigma_{i,t} \sigma_{j,t}} \]

- CIX is estimated using the implied volatility from the portfolio option, \( \sigma_{P,t} \), and the implied volatilities from options on each of the portfolio assets, \( \sigma_{i,t} \).

- Thus, options on the assets portfolio as well as options on the individual assets are required.

- For the calculation of CIX over a T-day horizon, implied volatilities should be derived by options with the same time-to-maturity, T.
CIX Interpretation

- An interesting property of CIX: 0 ≤ CIX ≤ 1

- Minimum CIX: $\sigma_{P,\text{min},t}^2 = \sum_{i=1}^{N} w_{i,t}^2 \sigma_{i,t}^2$

- Maximum CIX: $\sigma_{P,\text{max},t}^2 = \sum_{i=1}^{N} w_{i,t}^2 \sigma_{i,t}^2 + 2 \sum_{i=1}^{N-1} \sum_{j>i} w_{i,t} w_{j,t} \sigma_{i,t} \sigma_{j,t}$

- CIX represents how far lies the portfolio variance between the minimum variance (assuming zero correlations) and the maximum variance (assuming perfect correlations).

$$CIX_t = \frac{\sigma_{P,t}^2 - \sigma_{\text{Perf},t}^2}{\sigma_{\text{Perf},t}^2 - \sigma_{\text{Zero},t}^2}$$
Application of CIX to index options

This study provides an application of CIX to index options, since:

- Index options are widely traded in several international exchanges.

- Both stock and index prices are usually assumed to follow geometric Brownian motion for option valuation purposes.

- CIX calculated from an index option provides a measure of the market portfolio diversification in the specific market represented by the underlying index.
An implied correlation index for DJIA index (called DJCIX) is constructed based on equation (3).

We construct implied volatility indices for the DJIA and the constituent stocks (Whaley, 1993).

For each stock and the index, a synthetic ATM implied volatility with 30 calendar days to maturity is computed using 4 OTM options, 2 calls and 2 puts at the strike prices closest to the money and the 2 maturities closest to 30 calendar days.
Empirical Application: Methodology (2)

- A number of filtering rules were applied.
- We use the Black-Scholes (1973) and Merton (1973) option pricing model for the European options on DJIA index.
- We use the cash-dividend adjusted binomial method for the American stock options.
- The synthetic ATM volatilities were adjusted to a trading day basis.
- DJCIX represents the weighted average of all ATM implied pairwise correlations with constant maturity (22 trading days).
Data

Sample Period: October 2001 – October 2002

Option pricing model parameters:
- Option exercise price
- Option maturity
- Interest rates – London euro-currency bid/ask middle rates on US dollars (Datastream)
- Dividends – Cash dividends (Datastream)
- Underlying stock and index prices (Datastream)

Observed option prices
- Midpoints of last bid-ask quotes on DJIA options (traded in CBOE, AMEX, PHLX, ISE).
- Midpoints of last bid-ask quotes on stock options (traded in CBOE).
## Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max</th>
<th>Min</th>
<th>St.Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DJCIX</strong></td>
<td>0.4575</td>
<td>0.4419</td>
<td>0.8826</td>
<td>0.2927</td>
<td>0.0988</td>
<td>0.7296</td>
<td>3.6145</td>
<td>21.6199</td>
</tr>
<tr>
<td><strong>DJCIX changes</strong></td>
<td>0.0000</td>
<td>0.0006</td>
<td>0.4365</td>
<td>-0.4363</td>
<td>0.0777</td>
<td>0.0227</td>
<td>11.2517</td>
<td>584.4581</td>
</tr>
<tr>
<td><strong>DJVIX</strong></td>
<td>0.2784</td>
<td>0.2494</td>
<td>0.4822</td>
<td>0.1821</td>
<td>0.0770</td>
<td>0.6851</td>
<td>2.2440</td>
<td>21.1242</td>
</tr>
<tr>
<td><strong>DJVIC changes</strong></td>
<td>0.0000</td>
<td>-0.0014</td>
<td>0.0915</td>
<td>-0.1057</td>
<td>0.0274</td>
<td>0.2644</td>
<td>5.2088</td>
<td>44.2763</td>
</tr>
</tbody>
</table>

### Autocorrelations

<table>
<thead>
<tr>
<th></th>
<th>ρ(1)</th>
<th>ρ(2)</th>
<th>ρ(3)</th>
<th>ρ(4)</th>
<th>ρ(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DJCIX</strong></td>
<td>0.6917*</td>
<td>0.6353*</td>
<td>0.6255*</td>
<td>0.6018*</td>
<td>0.5286*</td>
</tr>
<tr>
<td><strong>DJCIX changes</strong></td>
<td>-0.4076*</td>
<td>-0.0748</td>
<td>0.0218</td>
<td>0.0807</td>
<td>-0.1747*</td>
</tr>
<tr>
<td><strong>DJVIX</strong></td>
<td>0.9365*</td>
<td>0.9051*</td>
<td>0.8725*</td>
<td>0.8477*</td>
<td>0.8277*</td>
</tr>
<tr>
<td><strong>DJVIC changes</strong></td>
<td>-0.2599*</td>
<td>0.0063</td>
<td>-0.0577</td>
<td>-0.0436</td>
<td>-0.1430*</td>
</tr>
</tbody>
</table>

### Cross Correlations

<table>
<thead>
<tr>
<th></th>
<th>DJIA log returns^a</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DJCIX changes</strong></td>
<td>-3</td>
</tr>
<tr>
<td><strong>DJVIX changes</strong></td>
<td>-0.0022</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>DJIA log returns^b</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DJCIX changes</strong></td>
<td>-3</td>
</tr>
<tr>
<td><strong>DJVIX changes</strong></td>
<td>-0.0050</td>
</tr>
</tbody>
</table>
Univariate properties

- The standard deviation of DJCIX levels and changes is higher than the standard deviation of DJVIX levels and changes, respectively.

- DJCIX ranges from 0.2927 (02/02) to 0.8826 (07/02).

- The distribution of DJCIX is positively skewed and leptokurtic.

- The normality assumption is rejected.
The first-order autocorrelation of DJCIX is significantly high, equal to 69%.

The first-order autocorrelation of DJCIX changes indicates a significant negative autocorrelation structure.

The autocorrelation structure of DJVIX is similar to the one reported on other volatility indices.
Cross-Correlations

- Significant negative cross-correlation coefficients between the DJCIX changes and the contemporaneous DJIA index returns.

- A significantly positive and large correlation between the implied correlation index changes and the current implied volatility index changes.

- While correlations tend to increase during high volatile periods, they tend to decrease before and after them.
Intraweek Behavior of DJCIX

- We regress the DJCIX changes on five day-of-the-week dummy variables:

\[ \Delta DJCIX_t = \sum_{k=1}^{5} \beta_k D_{k,t} + \epsilon_t \]

- Results indicate that no intraweek patterns exist in the DJCIX behavior.

<table>
<thead>
<tr>
<th>DJCIX changes</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.2666)</td>
<td>-0.0038</td>
<td>0.0182</td>
<td>-0.0056</td>
<td>-0.0044</td>
<td>-0.0011</td>
</tr>
<tr>
<td>(-0.2666)</td>
<td>(1.8024)</td>
<td>(-0.5981)</td>
<td>(-0.4386)</td>
<td>(-0.0735)</td>
<td></td>
</tr>
</tbody>
</table>
Stylized Facts About Correlation

- Correlation persistence and memory (Sheedy, 1998; Lundin et al., 1999; Andersen et al., 2001)

- Correlation asymmetry: correlation tends to increase when markets move down (Erb et al., 1994; Longin and Solnik, 1995; Cho and Engle, 1995; Andersen et al., 2001, Ang and Chen, 2002).

- Correlation increases in high volatile periods (Solnik et al., 1996, Ramchand and Susumel, 1998; Sheedy, 1998; Andersen et al. (2001).
Correlation Persistence and Memory (1)
Correlation Persistence and Memory (2)

- Correlation up to 22 lags could be attributed to the index construction.

- Autocorrelations remain significant up to 41 lags suggesting a long-run dependence in the implied correlation index.
Correlation Asymmetry (1)

We examine the correlation asymmetry with the following two regressions:

\[ \Delta DJCIX_t = \alpha + \sum_{i=1}^{5} \beta_i \Delta DJCIX_{t-i} + \sum_{j=-2}^{0} \gamma_j R_{t+j} + \sum_{j=-2}^{0} \gamma_{|j|} \left| R_{t+j} \right| + \varepsilon_t \]

\[ \Delta DJCIX_t = \sum_{i=1}^{5} \beta_i \Delta DJCIX_{t-i} + \gamma^+ D_t^+ R_t + \gamma^- D_t^- R_t \]
Correlation Asymmetry (2)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\gamma_0$</th>
<th>$\gamma_{-1}$</th>
<th>$\gamma_{-2}$</th>
<th>$\gamma_{0}$</th>
<th>$\gamma_{-1}$</th>
<th>$\gamma_{-2}$</th>
<th>$R^2$ adj.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1$^a$</td>
<td>-0.9088</td>
<td>-0.3586</td>
<td>-0.0896</td>
<td>0.6079</td>
<td>-0.2488</td>
<td>-0.1357</td>
<td>0.3258</td>
</tr>
<tr>
<td></td>
<td>(-2.3945)</td>
<td>(-1.4847)</td>
<td>(-0.3572)</td>
<td>(1.2563)</td>
<td>(-0.6444)</td>
<td>(-0.4555)</td>
<td></td>
</tr>
<tr>
<td>Model 2$^b$</td>
<td>$\gamma^+$</td>
<td>$\gamma^-$</td>
<td>$R^2$ adj.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.5893</td>
<td>-1.1833</td>
<td>0.3286</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.9842)</td>
<td>(-4.5260)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Correlation Asymmetry (3)

- The results from regression indicate a strong negative contemporaneous relationship between DJCIX changes and the DJIA index returns.

- No significant relationship exists between the lagged DJIA index returns and the correlation index changes.

- We find evidence of asymmetric responses of the correlation index changes to contemporaneous DJIA index returns.
Correlation and Volatility Co-movement (1)
Correlation and Volatility Co-movement (2)

- We investigate this relationship by conducting the following regression:

\[ \Delta DJCIX_t = \sum_{i=1}^{5} \beta_i \Delta DJCIX_{t-i} + \sum_{j=0}^{5} \delta_j \Delta DJVIX_{t-j} + \epsilon_t \]

- Results:

<table>
<thead>
<tr>
<th>Model 3c</th>
<th>( \delta_0 )</th>
<th>( \delta_1 )</th>
<th>( \delta_2 )</th>
<th>( \delta_3 )</th>
<th>( \delta_4 )</th>
<th>( \delta_5 )</th>
<th>( R^2 ) adj.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.0460</td>
<td>0.0146</td>
<td>-0.2814</td>
<td>0.1105</td>
<td>0.2594</td>
<td>-0.4670</td>
<td>0.72855</td>
</tr>
<tr>
<td></td>
<td>(7.8411)</td>
<td>(0.0710)</td>
<td>(-1.2706)</td>
<td>(0.5801)</td>
<td>(1.2595)</td>
<td>(-2.8476)</td>
<td></td>
</tr>
</tbody>
</table>

- Only the coefficients of the contemporaneous and the five lag DJVIX changes are significant.
Granger Causality tests (1)

- We test whether lagged DJVIX changes affect current DJCIX change and/or lagged DJCIX changes affect current DJVIX changes.

\[
\Delta DJCIX_t = \sum_{i=1}^{n} a_{1i} \Delta DJVIX_{t-i} + \sum_{i=1}^{n} \beta_{1i} \Delta DJCIX_{t-i} + \epsilon_{1t}
\]

\[
\Delta DJVIX_t = \sum_{i=1}^{n} a_{2i} \Delta DJCIX_{t-i} + \sum_{i=1}^{n} \beta_{2i} \Delta DJVIX_{t-i} + \epsilon_{2t}
\]

- \(H_0: a_{11} = \ldots = a_{1n} = 0\)

- \(H_0: \beta_{11} = \ldots = \beta_{1n} = 0\)
Granger Causality tests (2)

- Results

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>F-statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: a_{11} = \ldots = a_{1n} = 0$</td>
<td>1.4193</td>
<td>0.2190</td>
</tr>
<tr>
<td>$H_0: b_{11} = \ldots = b_{1n} = 0$</td>
<td>3.5251</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

While changes in the stock market volatility do not affect future changes in the level of stock return correlations, changes in the level of stock return correlations appear to cause future stock market volatility increases/decreases.
Forecasting Ability of CIX

- Our objective is to evaluate the performance of CIX as a forecast of future realized correlation index.

- We test whether CIX is an unbiased forecast of future realized average pair-wise correlation and its forecast error is orthogonal to the market’s information set.

- We measure realized CIX using 5-minute returns for the DJIA index and the constituent stocks.

- We use a historical correlation index for comparison.
We estimate the parameters $\alpha$ and $\beta$ of the following equation using GMM:

$$CIX_t^* - CIX_{t-1} = \alpha + \beta (CIX_t - CIX_{t-1}) + \varepsilon_t$$

We conduct a Wald test of the joint null that the estimates of $\alpha$ and $\beta$ are equal to zero and one, respectively.
Orthogonality Test

- The orthogonality of CIX is investigated relative the historical correlation index, and vice versa:

\[ CIX_t^* = \alpha + \beta CIX_t + \varepsilon_t \]

- We test the orthogonality of the forecast using Hansen’s (1982) test of overidentifying restrictions.
## Test Results

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\alpha}$</th>
<th>$t_\alpha$</th>
<th>$\hat{\beta}$</th>
<th>$t_\beta$</th>
<th>$R^2$ adj.</th>
<th>$CS^2$</th>
<th>$OI^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unbiasedness Tests</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Implied CIX</td>
<td>-0.1737</td>
<td>-15.56</td>
<td>0.5016</td>
<td>-44.13</td>
<td>0.2854</td>
<td>1948.70</td>
<td></td>
</tr>
<tr>
<td>Historical CIX</td>
<td>0.0800</td>
<td>2.15</td>
<td>0.6187</td>
<td>-1.8200</td>
<td>0.0077</td>
<td>6.78</td>
<td></td>
</tr>
<tr>
<td><strong>Orthogonality Tests</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Implied CIX</td>
<td>0.0137</td>
<td>0.41</td>
<td>0.5853</td>
<td>-5.63</td>
<td>0.4786</td>
<td>234.92</td>
<td>0.06</td>
</tr>
<tr>
<td>Historical CIX</td>
<td>0.3051</td>
<td>8.05</td>
<td>-0.307</td>
<td>-6.93</td>
<td>0.3360</td>
<td>516.85</td>
<td>5.08</td>
</tr>
</tbody>
</table>
Conclusions (1)

- We propose a new measure, the average implied correlation index, that provides a market’s forecast of the future average correlation between the underlying assets in an index.

- The notion of the implied correlation index offers both academics and practitioners a new measure of market portfolio diversification.

- The implied correlation is well-behaved and varies substantially over time.
Conclusions (2)

- Several stylized facts reported in the literature about correlation are also present in the DJCIX dynamics.

- Correlations tend to increase in bear markets and in high volatile markets decreasing the benefits of diversification when they are needed most.

- CIX is an upward biased forecast of the realized correlation index. However, it exhibits a stronger relationship to future realized correlation compared with a historical estimate.